# Convex-Arc Drawings of Pseudolines

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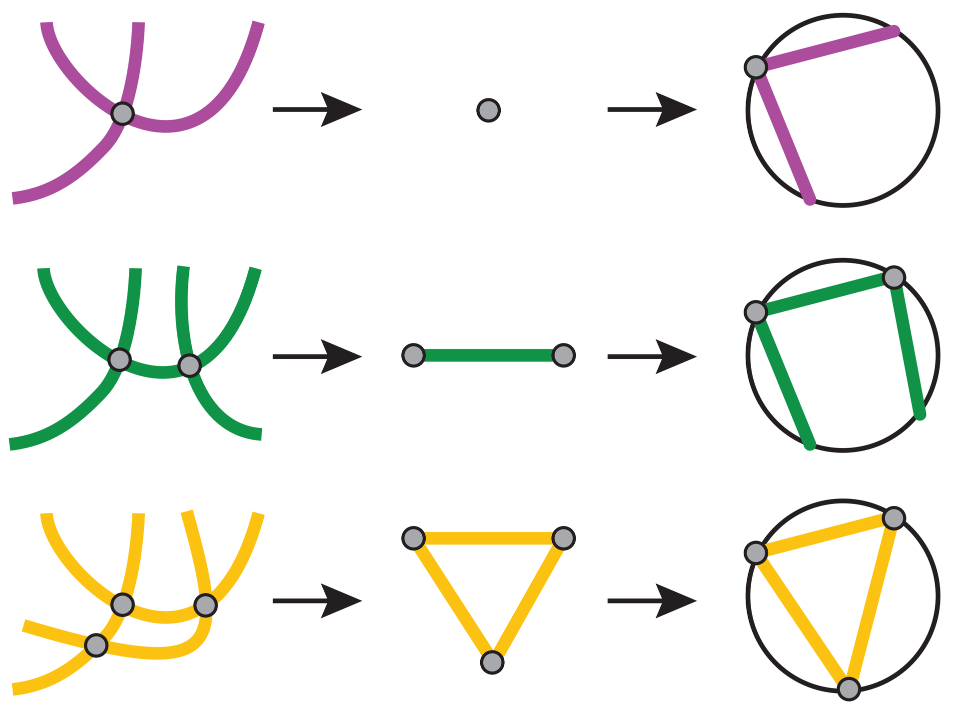
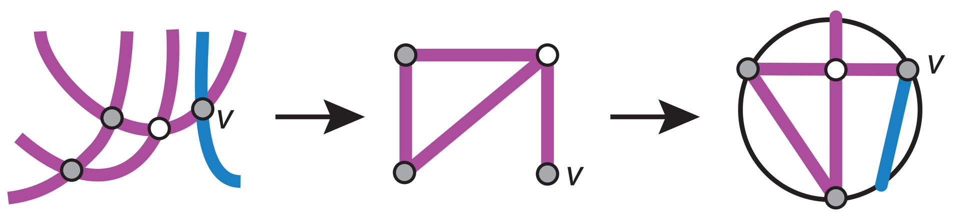
## Weak, outerplanar arrangements

**Theorem 1.** *Every weak, outerplanar pseudoline arrangement may be represented by a set of chords of a circle.*

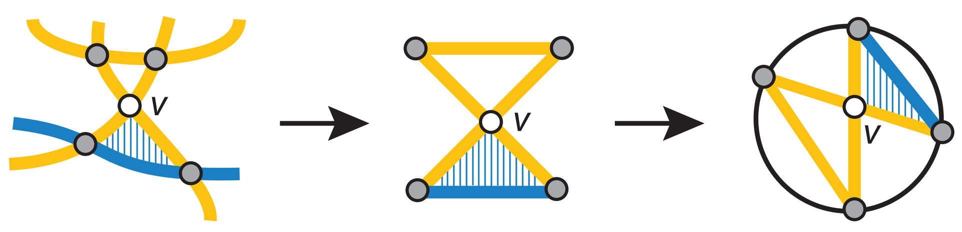
**1.** If the graph contains an isolated vertex, edge, or triangle, it can be represented directly.



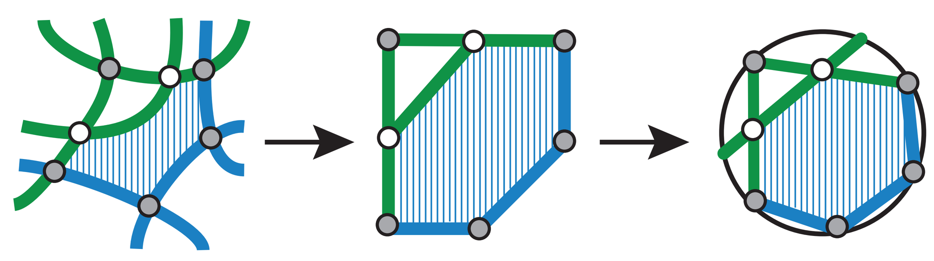
**2.** If the graph contains a vertex *v* of degree 1, removing it gives us an arragement with one fewer crossing. In the chord representation of this arrangement, move the neighbour of *v* inside the circle. Now *v* can be added to the representation by adding a new chord, crossing the chords on which *v* should lie close to the circle.



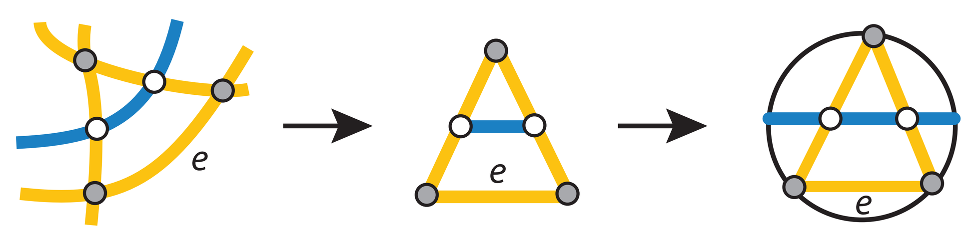

**3.** If the graph contains a triangular face which is attached to the rest of the graph by a single articulation vertex *v*, removing the pseudoline forming the edge opposite from *v* gives us an arrangement with two fewer crossings. In the chord representation of this arrangement, move *v* to the inside of the circle. The triangular face can now be added by inserting an extra chord.



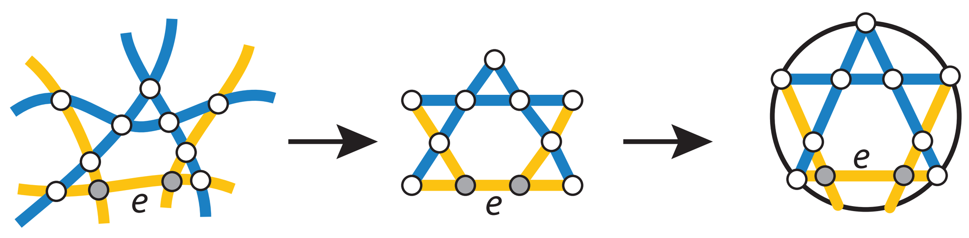

**4.** If the graph contains a bounded face with more than three edges, at most one of which is also adjacent to another bounded face, remoing the vertices not on the shared edge gives us an arrangement with fewer crossings. Move the endpoints of the shared edge inside the circle and close the face by adding a chain of new chords.

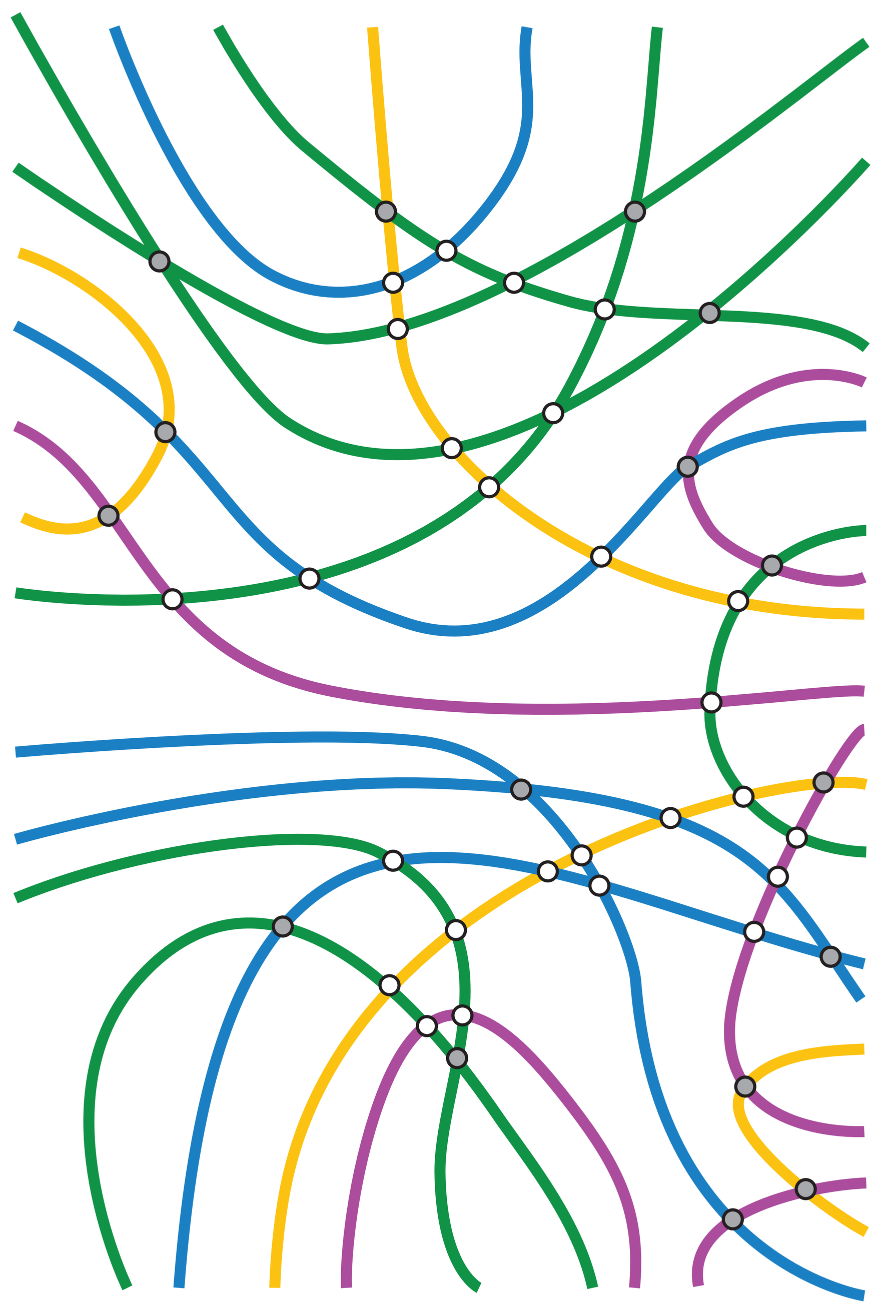


**5.** If the graph contains an interior edge *e* with a quadrilateral and one or two triangles on one side of it, removing the pseudoline forming the edge of the quadrilateral opposite of *e* gives an arrangement with fewer crossings. We can represent the original arrangement by adding an extra chord to the chord representation of this simpler arrangement.

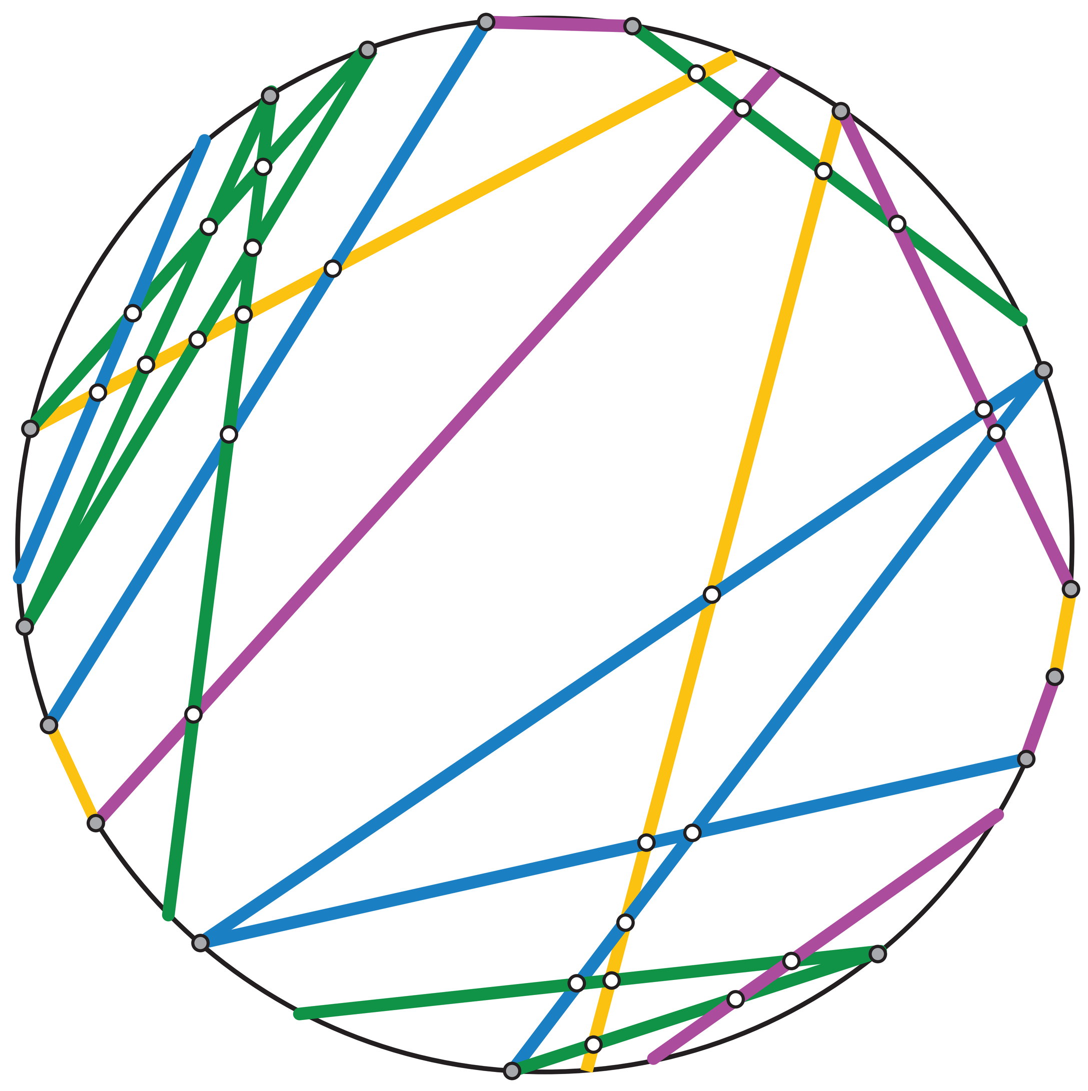



**6.** In the remaining case, an interior edge *e* bounds a face *f* with four or more sides, such that the only other faces or edges on that side of *e* are triangles that share an edge with *f*. Removing all pseudolines that touch *f* but not *e* yields a simple arrangement. We can represent the original arrangement by adding the removed pseudolines back.



Pseudoline arrangement

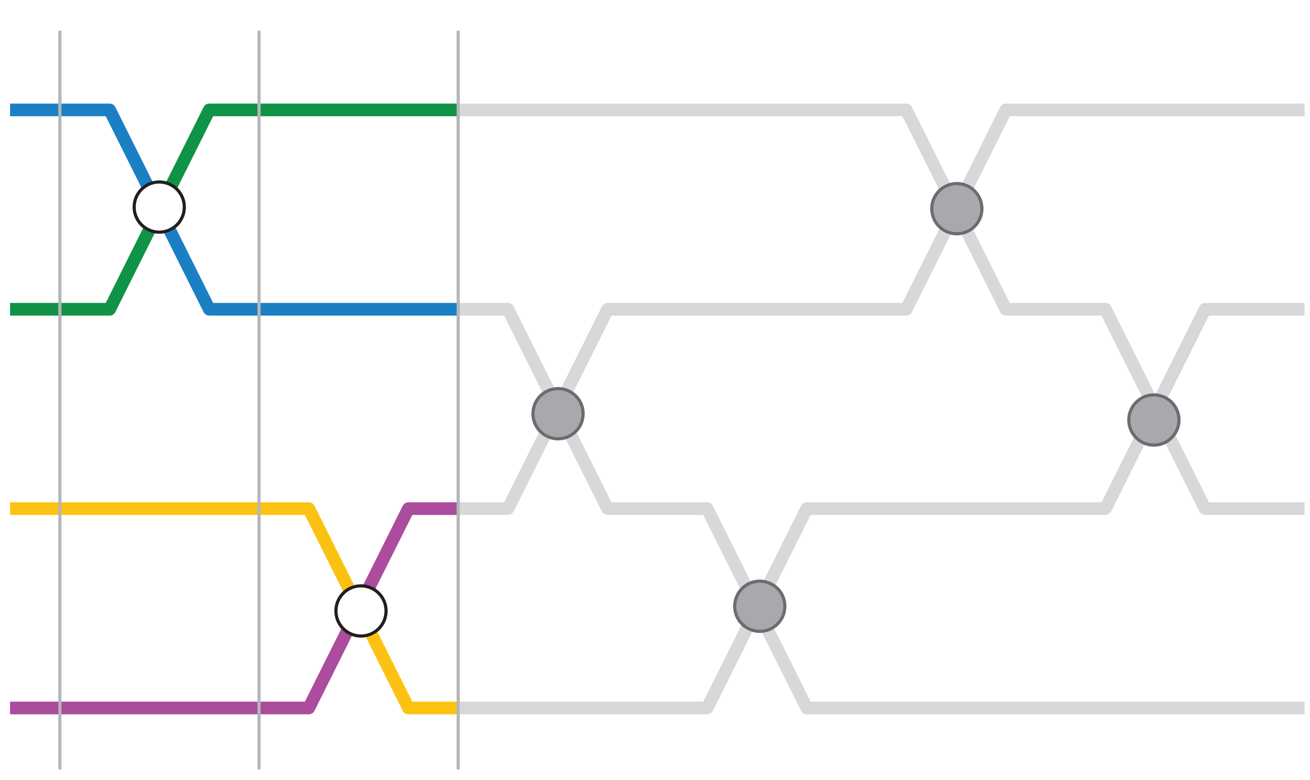


Chord representation

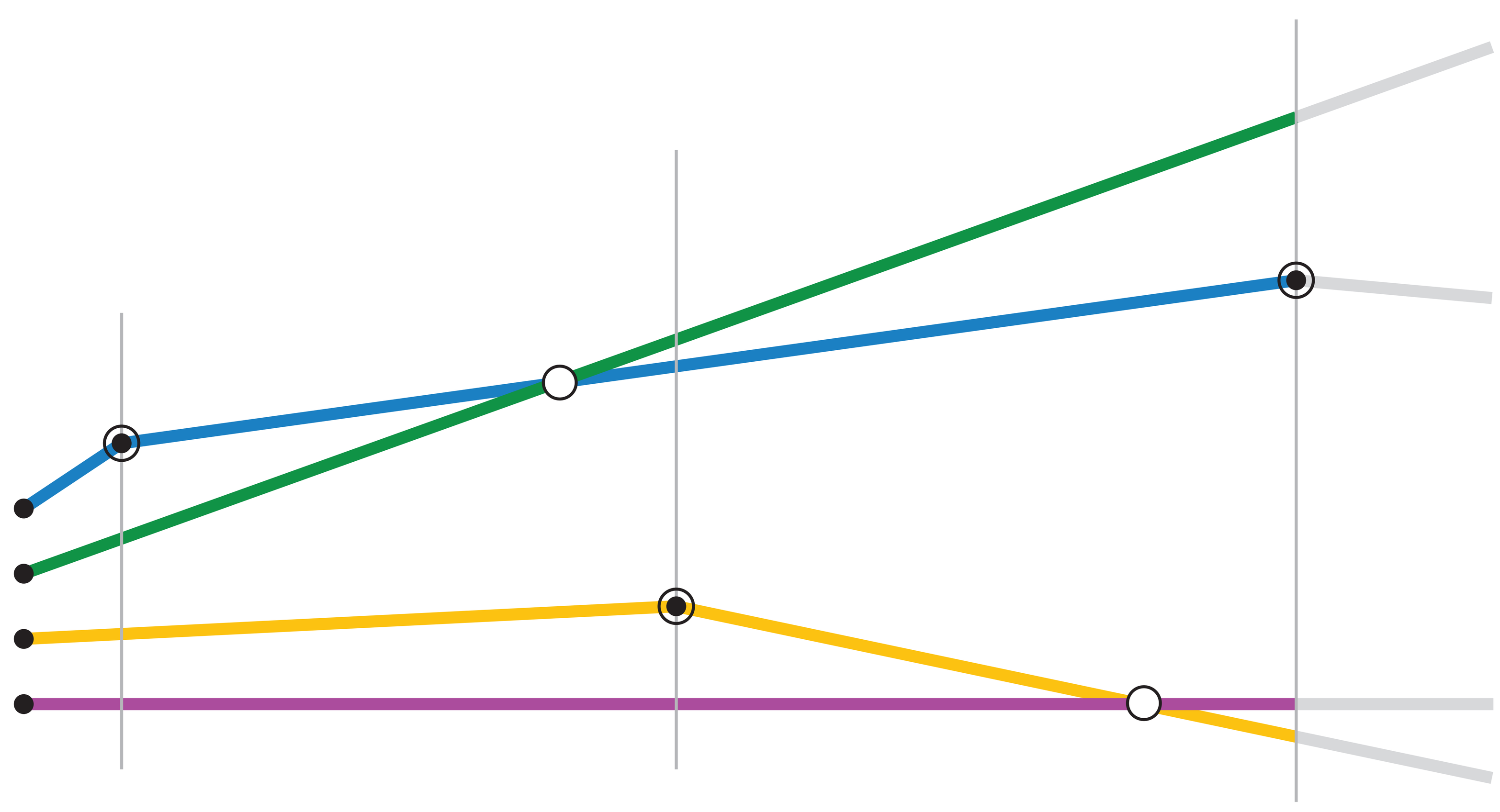
**Corollary.** *Every weak, outerplanar pseudoline arrangement may be represented by convex polygonal chains with only two bends.*

## General arrangements

**Theorem 2.** *Every n-element pseudoline arrangement can be drawn with convex polylines, each of complexity at most n.*



Wiring diagram



Polyline representation

**1.** Represent the pseudoline arrangement as a wiring diagram.

**2.** For the polyline representation initially represent each pseudoline as an infinite ray. The pseudolines start out in the same top-to-bottom order as in the wiring diagram, each pseudoline has a higher slope than any of the pseudolines below it.

**3.** Sweep the wiring diagram from left to right. Whenever two pseudolines need to cross, they are already adjacent in the representation. We make the higher one of the two bend downwards, such that its slope becomes smaller than that of the pseudoline it needs to cross, but still larger than the ones below that.

**Theorem 3.** *There exist simple arrangements of n pseudolines that, when represented by polygonal chains, require some pseudolines to have $\Omega(n)$ bends.*